Indian Statistical Institute
First Semester Back Paper Examination 2004-2005
B.Math (Hons.) I Year
Analysis I

Time: 3 hrs Maximum Marks: 100

1. Let $f : (0,1) \to I\!R$ be a continuous function such that $\lim_{x \to 0+} f(X) = 0$ and $\lim_{x \to 1-} f(x) = 1$. Show that $\exists \ x_0 \in (0,1)$ such that $f(x_0) = \frac{\sqrt{3}}{2}$ [15]

2. Let $g : [0,1] \to I\!R$ be a continuous function. Suppose that for each $x \in [0,1], \quad \exists y \in [0,1]$ such that
$$|g(y)| \leq |g(x)|$$
Show that $\exists \ x_0 \in [0,1]$ such that $g(x_0) = 0$. [15]

3. Let $f : [a,b] \to I\!R$ be a continuous function and let $\alpha : [a,b] \to I\!R$ be an increasing function. Show that $f$ is Riemann-Stierltjis integrable with respect to $\alpha$. [20]

4. Let $g : I\!R \to I\!R$ be a non-zero function satisfying
$$g(x+y) = g(x)g(y) \quad \forall x, y \in I\!R$$
. Show that if $g$ is continuous at 0 it is continuous everywhere. Further show that if $g$ is continuous then $g(x) = b^x$ for some $b \in I\!R$. [20]

5. Show that countable union of countable sets is countable. [10]

6. $A, B$ be non-empty subsets of $I\!R$ which are bounded above.
Take $S = \{a \in b : \quad a \in A, \ b \in B\}$
Then show that $\sup(S) = \ \sup(A) + \sup(B)$. [10]

7. Let $h : I\!R \to I\!R$ be a function satisfies
$$|h(x) - h(y)| \leq R(x-y)^2$$
for some $R \in I\!R$. Then show that $h$ must be a constant function. [10]